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MATHEMATICAL MODELING OF THE BOREHOLE HEATING PROCESS BY MEANS OF AXIAL PLASMATRON

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Abstract. The article presents a mathematical model that allows determining the main parameters of the plasma-dynamic coolant jet in the process of thermal heating of the borehole inner surface. The mathematical model of low-temperature plasma motion along the wellbore consists of the k-ε turbulence model equations, the continuity and energy equations for the gas flow, and the non-stationary heat conduction equation for calculating the temperature of a cylindrical flange pipe, which models the rock mass around the borehole. The equations are written in a cylindrical coordinate system for the radial and longitudinal components of the velocity of a low-temperature plasma flow. The differential equations of the mathematical model were supplemented with the corresponding initial and boundary conditions. The initial conditions were the known gas temperatures in the borehole and the initial temperature of the cylindrical flange pipe. The boundary conditions, in addition to the corresponding relations for the turbulence model, were the known parameters of the plasma flow at the inlet to the cylindrical pipe and the conditions for stabilization of the flow at the outlet. No-slip conditions for the flow and boundary conditions of the third order for the energy equation and the heat equation were used on the fixed boundary of the flanged pipe. To calculate the equations of the mathematical model, the numerical finite element method was used. The adequacy of the model of the borehole heating process by the plasma flow was verified by comparing the numerical calculation with experimental data. Experimental data confirm the adequacy of the proposed mathematical model. The difference between numerical and experimental data does not exceed 4.1%. The proposed mathematical model can be used to calculate the temperature of the inner surface of the borehole before it is chipped during heating.

Keywords: mathematical model, borehole, low temperature plasma flow, pipe heating.

Introduction. A problem of mathematical modeling of the process of the rock spallation reaming is urgent as well and it is described in several publications [1-3].

The latest domestic experimental and theoretical researches of the problems of crystalline structures destruction by plasma are known [4, 5].

Possibilities of analytical determination of optimal parameters of thermal influence on rocks are limited by solution of thermoelasticity equations and contact tasks of durability theory.

Such a task formulation is unacceptable due to complication of taking into account of substantial change of physical and thermophysical rock properties in the processes of its heating and mechanical loading.

Due to an existence of fundamental differences among the results of the known publications and limitations of investigational parameters of heat transfer medium that interacts with the borehole surface, it is obvious necessity of mathematical model development that allows to define basic plasmadynamic jet parameters of heat transfer medium in the process of borehole thermal heating.

Methods. Mathematical model of the plasma motion along the borehole consists of the k-ε turbulence model equations as follows [6, 7]:

- equations of plasma motion and continuity equation:

$$\begin{aligned}&\frac{\partial(\rho_{pl}U_r)}{\partial\tau}+\frac{\partial(\rho_{pl}\cdot rU_rU_r)}{r\partial r}+\frac{\partial(\rho_{pl}\cdot U_xU_r)}{\partial x}=\\&=\frac{\partial}{\partial r}\left[(\mu_{pl}+\mu_m)\left(\frac{\partial U_r}{\partial r}+\frac{U_r}{r}\right)\right]+\frac{\partial}{\partial x}\left[(\mu_{pl}+\mu_m)\frac{\partial U_r}{\partial x}\right]-\frac{\partial P_{pl}}{\partial r},\end{aligned}\tag{1}$$

$$\begin{aligned}&\frac{\partial(\rho_{pl}U_x)}{\partial\tau}+\frac{\partial(\rho_{pl}\cdot rU_rU_x)}{r\partial r}+\frac{\partial(\rho_{pl}\cdot U_xU_x)}{\partial x}=\\&=\frac{\partial}{r\partial r}\left[(\mu_{pl}+\mu_m)r\frac{\partial U_x}{\partial r}\right]+\frac{\partial}{\partial x}\left[(\mu_{pl}+\mu_m)\frac{\partial U_x}{\partial x}\right]-\frac{\partial P_{pl}}{\partial x},\end{aligned}\tag{2}$$

$$\frac{\partial(\rho_{pl}U_r)}{\partial r}+\frac{\partial(\rho_{pl}U_x)}{\partial x}+\frac{\rho_{pl}U_r}{r}=0;\tag{3}$$

- eddy viscosity:

$$\mu_t=C_\mu\rho_{pl}\frac{k^2}{\varepsilon};\tag{4}$$

- equations for turbulence kinetic energy and turbulence kinetic energy dissipation rate:

$$\begin{aligned}&\frac{\partial(\rho_{pl}k)}{\partial t}+\frac{1}{r}\frac{\partial(\rho_{pl}rU_rk)}{\partial r}+\frac{\partial(\rho_{pl}U_xk)}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left[\left(\mu_{pl}+\frac{\mu_t}{\sigma_k}\right)r\frac{\partial k}{\partial r}\right]+\\&+\frac{\partial}{\partial x}\left[\left(\mu_{pl}+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x}\right]+G-\rho\varepsilon\end{aligned},\tag{5}$$

$$\begin{aligned}&\frac{\partial(\rho_{pl}\varepsilon)}{\partial t}+\frac{1}{r}\frac{\partial(\rho_{pl}rU_r\varepsilon)}{\partial r}+\frac{\partial(\rho_{pl}U_x\varepsilon)}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left[\left(\mu_{pl}+\frac{\mu_t}{\sigma_\varepsilon}\right)r\frac{\partial \varepsilon}{\partial r}\right]+\\&+\frac{\partial}{\partial x}\left[\left(\mu_{pl}+\frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x}\right]+C_{\varepsilon1}\frac{\varepsilon}{k}G-C_{\varepsilon2}\rho_{pl}\frac{\varepsilon^2}{k}\end{aligned},\tag{6}$$

$$G=\mu_t\left\{2\left[\left(\frac{\partial U_r}{\partial r}\right)^2+\left(\frac{U_r}{r}\right)^2+\left(\frac{\partial U_x}{\partial x}\right)^2\right]+\left(\frac{\partial U_x}{\partial r}+\frac{\partial U_r}{\partial x}\right)^2\right\},\tag{7}$$

where ρ_{pl} – plasma density; U_r and U_x – velocity components; μ_{pl} – plasma dynamic viscosity; P_{pl} – plasma pressure; k – turbulence kinetic energy; ε – turbulence kinetic energy dissipation rate.

Constants in equations (4)-(7) are as follows: C_μ=0.09, $C_{\varepsilon 1}$=1.44, $C_{\varepsilon 2}$=1.92, σ_k =1, σ_ε=1.3.

Governing equation of the heat transfer between plasma flow and internal surface of the flanged branch pipe is as follows:

$$\rho_{pl}c_{pl}\frac{\partial T_{pl}}{\partial \tau}+\frac{\partial\left(U_x\rho_{pl}c_{pl}T_{pl}\right)}{\partial x}+\frac{\partial\left(rU_r\rho_{pl}c_{pl}T_{pl}\right)}{r\partial r}=\frac{\partial}{r\partial r}\cdot\left(r\lambda_{pl}\frac{\partial T_{pl}}{\partial r}\right)+\frac{\partial}{\partial x}\cdot\left(\lambda_{pl}\frac{\partial T_{pl}}{\partial x}\right). \quad (8)$$

Governing equation of heat conduction in the flanged branch pipe is as follows:

$$\rho_{fb}\cdot c_{fb}\cdot\frac{\partial T_{fb}}{\partial \tau}=\frac{\partial}{r\partial r}\cdot\left(r\lambda_{fb}\frac{\partial T_{fb}}{\partial r}\right)+\frac{\partial}{\partial x}\cdot\left(\lambda_{fb}\frac{\partial T_{fb}}{\partial x}\right), \quad (9)$$

where c_{pl} and c_{fb} – specific heat capacity of plasma and material of the flanged branch pipe respectively; T_{pl} and T_{fb} – temperature of plasma and material of the flanged branch pipe respectively; λ_{pl} and λ_{fb} – thermal conductivity coefficient of plasma and material of the flanged branch pipe respectively.

Initial conditions:

- initial temperature of the medium inside the flanged branch pipe $T_{pl}\big|_{\tau=0}=303$ K;
- initial temperature of the flanged branch pipe $T_{fb}\big|_{\tau=0}=303$ K.

Boundary conditions for this problem are as follows (see Figure 1):

Axial symmetry boundary conditions are defined for boundary 1 [8]:

- for equations (1)-(3):

$$\frac{\partial U}{\partial n}=0; \quad (10)$$

- for equation (5) and equation (6):

$$\frac{\partial k}{\partial n}=0, \quad (11)$$

$$\frac{\partial \varepsilon}{\partial n}=0; \quad (12)$$

- for equation (8):

$$-\lambda_{pl} \cdot \frac{\partial T_{pl}}{\partial n} = 0. \tag{13}$$

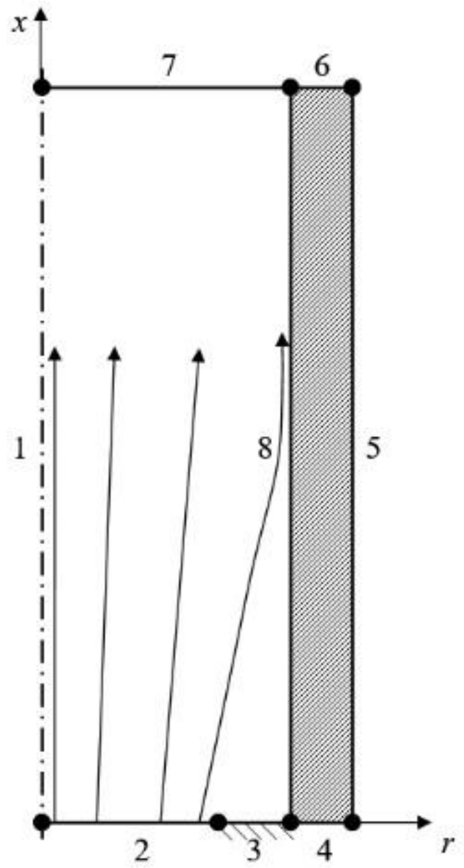

Figure 1 – Boundary conditions

For boundary 2:

- for equations (1)-(3) at the inlet of the flanged branch pipe for the plasma flow velocity components for r and x directions (U_r and U_x) as well as static pressure P are defined [9];

- for equation (5) and equation (6):

$$k = \frac{3}{2} \cdot \left(|U_x| \cdot I_t\right)^2; \tag{14}$$

$$\varepsilon = \frac{C_\mu^{0,75} \cdot k^{1,5}}{l_t}, \tag{15}$$

where l_t – turbulent length scale, that is defined as follows: $l_t = 0.07 \cdot d_{fb}$; I_t – turbulence intensity, that equals 5 %; where d_{fb} – diameter of the flanged branch pipe;

- for equation (8) at the inlet of the flanged branch pipe the plasma flow temperature is defined T_{pl}.

For boundary 7:

- for equations (1)-(3) at the outlet of the flanged branch pipe for the plasma flow static pressure *P* that equals to the atmospheric pressure is defined [9];
- for equation (5) and equation (6):

$$\frac{\partial k}{\partial n} = 0; \tag{16}$$

$$\frac{\partial \varepsilon}{\partial n} = 0; \tag{17}$$

- for equation (8):

$$-\lambda_{pl} \cdot \frac{\partial T_{pl}}{\partial n} = 0. \tag{18}$$

For boundary 3 and boundary 8:
- for equations (1)-(3):

$$\frac{\partial U}{\partial n} = 0; \tag{19}$$

$$v^{+} = \frac{1}{\kappa} \cdot ln\left(\frac{\delta_w}{l^{*}}\right) + C^{+}, \tag{20}$$

where κ – the Karman's constant, that equals κ=0.42; C^+ – a universal constant for smooth walls, that equals C^+=5.5; δ_w – distance from the wall (from the boundary 8); l^* – the viscous length scale, that is defined as follows [8]:

$$l^{*} = \frac{\mu_{pl}}{\rho_{pl} \cdot U_x}; \tag{21}$$

$$\delta_w^{+} = \frac{\delta_w}{l^{*}} = \frac{\rho_{pl} \cdot C_{\mu}^{0,25} \cdot k^{0,5} \cdot \delta_w}{\mu_{pl}} = 100; \tag{22}$$

- for equation (5) and equation (6):

$$\frac{\partial k}{\partial n} = 0; \tag{23}$$

$$\varepsilon = \frac{C_{\mu}^{0,75} \cdot k^{1,5}}{\kappa \cdot \delta_w}; \tag{24}$$

- for equation (8):

$$\varepsilon_{pf} \cdot \sigma_0 \cdot \left(T_{pl}^4 - T_{fb}^4\right) + \alpha_{fb} \cdot \left(T_{pl} - T_{fb}\right) = -\lambda_{fb} \cdot \frac{\partial T_{fb}}{\partial n}, \tag{25}$$

where ε_{pf} – the emissivity coefficient; σ_0 – Stefan-Boltzmann constant; α_{fb} – heat transfer coefficient from the plasma to the flanged branch pipe internal surface;

For boundaries 4, 5 and 6:

- for equation (9):

$$-\lambda_{fb} \cdot \frac{\partial T_{fb}}{\partial n} = \alpha_{amb} \cdot \left(T_{fb} - T_{amb}\right), \tag{26}$$

where α_{amb} – heat transfer coefficient from internal surface of the flanged branch pipe to ambient air; T_{amb} – temperature of ambient air.

For boundary 8:

- for equation (9):

$$-\lambda_{fb} \cdot \frac{\partial T_{fb}}{\partial n} = \varepsilon_{pf} \cdot \sigma_0 \cdot \left(T_{pl}^4 - T_{fb}^4\right) + \alpha_{fb} \cdot \left(T_{pl} - T_{fb}\right). \tag{27}$$

Equations (1)-(3), (5), (6), (8), (9) were solved via Galerkin method based on the finite element method. The triangular finite element discretization of the domain was carried out.

System of equations of the standard form is written [10, 11]:

$$[K] \cdot \{\Phi\} + \{F\} = 0, \tag{28}$$

where $[K]$ – matrix of the shape functions for all nodes of the finite elements; $\{\Phi\}$ – matrix of the functions that should be defined for all nodes of the two-dimensional domain; $\{F\}$ – matrix that comprises forces per unit volume of the plasma flow as well as boundary conditions.

Matrix could contain an integral of the volume forces as well as boundary conditions.

Results and discussion. Model adequacy checking of the borehole heating process by means of the plasma flow was made by comparison of the numerical calculation with experimental data. Experimental study technique was described in paper [12].

Comparison of the temperature of the inner surface of the flanged branch pipe is shown in Figure 2.

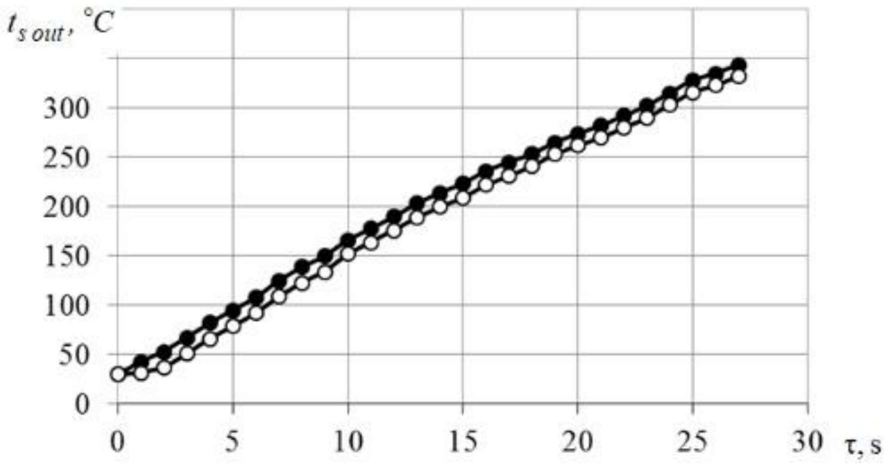

Figure 2 – Comparison of the temperature of the inner surface of the flanged branch pipe

Experimental data confirm an adequacy of the proposed mathematical model. The difference between numerical and experimental data does not exceed 4.1 %.

Conclusion. The proposed mathematical model could be used for calculation of the temperature of the borehole inner surface prior to its spallation during the heating process.

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ПРОЦЕСУ НАГРІВАННЯ СВЕРДЛОВИНИ ЗА ДОПОМОГОЮ АКСІАЛЬНОГО ПЛАЗМОТРОНУ

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Анотація. У статті наведено математичну модель, яка дозволяє визначити основні параметри плазмодинамічного потоку теплоносія у процесі теплового нагрівання внутрішньої поверхні свердловини. Математична модель руху низькотемпературної плазми вздовж отвору свердловини складається з рівнянь k-ε моделі турбулентності, рівняння нерозривності та енергії для газового потоку та нестаціонарного рівняння теплопровідності для розрахунку температури циліндричного фланцевого патрубка, який моделює гірський масив навколо свердловини. Рівняння записані в циліндричній системі координат для радіальної та поздовжньої складової швидкості низькотемпературного плазмового потоку плазми. Диференціальні рівняння математичної моделі доповнювалися відповідними початковими та граничними умовами. Початковими умовами були відомі температури газу в свердловині та початкова температура фланцевого циліндричного патрубка. Граничними умовами, крім відповідних співвідношень моделі турбулентності, були відомі параметри плазмового потоку на вході в циліндричний патрубок і умови стабілізації потоку на виході. На нерухомій межі фланцевого патрубка використовувалися умови прилипання для потоку та граничні умови третього роду для рівняння енергії та рівняння теплопровідності. Для розрахунку рівнянь математичної моделі використовувався чисельний метод кінцевих елементів. Перевірка адекватності моделі процесу нагрівання свердловини потоком плазми проводилася шляхом порівняння чисельного розрахунку експериментальними даними. Експериментальні дані підтверджують адекватність запропонованої математичної моделі. Різниця між чисельними та експериментальними даними не перевищує 4,1 %. Запропонована математична модель може бути використана для розрахунку температури внутрішньої поверхні свердловини до її сколювання в процесі нагрівання.

Ключові слова: математична модель, свердловина, потік низькотемпературної плазми, нагрівання патрубка.

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